International Risk Sharing and the Transmission of Productivity Shocks

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This paper shows that standard international business cycle models can be reconciled with the empirical evidence on the lack of consumption risk sharing. First, we show analytically that with incomplete asset markets productivity disturbances can have large uninsurable effects on wealth, depending on the value of the trade elasticity and shock persistence. Second, we investigate these findings quantitatively in a model calibrated to the U.S. economy. With the low trade elasticity estimated via a method of moments procedure, the consumption risk of productivity shocks is magnified by high terms of trade and real exchange rate (RER) volatility. Strong wealth effects in response to shocks raise the demand for domestic goods above supply, crowding out external demand and appreciating the terms of trade and the RER. Building upon the literature on incomplete markets, we then show that similar results are obtained when productivity shocks are nearly permanent, provided the trade elasticity is set equal to the high values consistent with micro-estimates. Under both approaches the model accounts for the low and negative correlation between the RER and relative (domestic to foreign) consumption in the data—the “Backus–Smith puzzle”.

1. INTRODUCTION

Is consumption risk optimally hedged across countries? Despite the development of international financial markets in the last decades, the answer from a large body of financial and macroeconomic research appears to be “no”.1 While the literature has analysed many different facets of (the lack of) international risk sharing, a crucial testable implication is that, in a world economy characterized by large deviations from purchasing power parity (PPP), domestic households should consume more when their consumption basket is relatively cheap. As first shown by Backus and Smith (1993), this is clearly at odds with the data. For most OECD countries, the correlation between relative consumption and the RER (i.e. the relative price of consumption across countries) is generally low, and even negative. A striking illustration of such finding is presented in Figure 1, which plots annual U.S. consumption relative to the other OECD countries and the U.S. real trade-weighted exchange rate in the period 1973–2002, both in deviations from an Hodrick–Prescott (HP) filtered trend. The swings in the dollar in real terms are not associated with movements of the consumption ratio in the same direction; on the contrary, the two variables tend to comove negatively.

1. See the surveys by Lewis (1999) and Obstfeld and Rogoff (2001).
An essential element in explaining a low degree of international risk sharing is that international financial markets are not developed enough. Yet, the literature offers convincing arguments to doubt that incomplete asset markets *per se* be sufficient to bring models in line with the Backus–Smith evidence on (the lack of) risk sharing. Several contributions have shown that the equilibrium allocation in economies that only trade in international, uncontingent bonds may be quite close to the first best, provided shocks are not permanent (*e.g.* see Baxter and Crucini, 1993). Indeed, trade in bonds insures that the real rate of currency depreciation and the growth rate of relative consumption are highly and positively correlated in expectations—although not necessarily period by period *ex-post*, as is the case when markets are complete. In addition, Cole and Obstfeld (1991) has called attention to the potential role of movements in the terms of trade in insuring against production risk independently of trade in assets. This is because, regardless of technological spillovers, the international transmission of shocks via relative prices propagates the benefits of country-specific gains abroad, via relative price changes.

In this paper, we show that standard incomplete-market models of the international business cycle can be reconciled with the evidence on the lack of risk sharing. We focus our analysis on the general-equilibrium link between international price movements, relative wealth, and international consumption risk sharing—the Backus–Smith puzzle. Our contribution is twofold. First, in a simple two-country endowment economy we show analytically that under incomplete markets the correlation between relative consumption and the RER can have either sign, depending on the value of the price elasticity of tradables, and, when international trade in bonds is allowed, on the dynamics of endowment shocks. This result is important because it demonstrates that standard open-economy models are not fundamentally at odds with the Backus–Smith puzzle. Second, we explore the quantitative implications of our analytical findings in a model similar to Stockman and Tesar (1995), but in which asset markets are incomplete. The model features traded and non-traded goods, as well as home bias in domestic spending on tradables. In addition, as in Burstein, Neves and Rebelo (2003), we introduce distribution services produced with the intensive use of local inputs. Because of these features, the model generates realistic departures from PPP (*see, for example*, Burstein, Eichenbaum and Rebelo, 2005; and Crucini, Telmer and Zachariadis, 2005). In this setting, we reconsider the international transmission of productivity shocks and quantify two mechanisms whereby equilibrium wealth effects in response to these shocks can actually induce international price movements that hinder risk sharing.
In light of our analytical results, the quantitative ability of international business cycle models to generate strong wealth effects and low risk sharing depends on the parameterization of the technology shocks process and the value of the trade elasticity. We follow two distinct approaches. First, in line with the literature (e.g. Stockman and Tesar, 1995), we estimate the shock process from sectoral Solow residuals. Given the uncertainty in the literature regarding the trade elasticity, we estimate the latter via a method of moments procedure. Under this estimate, we find that terms of trade and RER volatility in response to productivity shocks result in large, uninsurable effects on relative wealth. In other words, large swings in international prices magnify the consumption risk of fundamental supply shocks, generating large departures from efficient risk sharing. As a result, the predicted correlation between the RER and relative consumption is negative. The estimated price elasticity of tradables is slightly below 1/2—a value within the range of estimates found in the macroeconomic literature (e.g. see Hooper, Johnson and Marquez, 2000). With our estimated trade elasticity, international consumption spillovers are negative: following a productivity increase, the home terms of trade and the RER appreciate, hurting foreign consumers. Such pattern of international transmission is due to non-trivial general equilibrium effects arising from market incompleteness. Because of home bias in consumption, domestic tradables are mainly demanded by domestic households. With a low price elasticity, a terms-of-trade depreciation that reduces domestic wealth relative to the rest of the world would actually result in a drop of the world demand for domestic goods—in equilibrium, the negative wealth effect in the home country would more than offset any global positive substitution and wealth effect. Therefore, for the world markets to clear, a larger supply of domestic tradables must be matched by an appreciation of the country’s terms of trade, driving up domestic wealth and demand. While accounting for the Backus–Smith puzzle, our economy also generates comovements in aggregates across countries, which are broadly in line with the evidence.

The value of the trade elasticity selected by the method of moments procedure is low, especially relative to micro-estimates and trade estimates. For instance, Bernard, Eaton, Jensen and Kortum (2003) finds a value as high as 4. With high trade elasticities, our analytical results suggest that the Backus–Smith puzzle can be matched if shocks are very persistent. This finding raises an important issue in light of the literature on incomplete markets, which has long made clear that productivity shocks close to a unit root can drive substantial wedges between allocations with complete and incomplete markets (see Baxter and Crucini, 1995; and Heathcote and Perri, 2002 among others). Is the same degree of shock persistence also sufficient to generate a negative correlation between relative consumption and the RER? We show that this is indeed the case, by carrying out a second set of experiments in which we set the trade elasticity equal to 4 and assume that the autoregressive coefficient of productivity shocks approach unity, similarly to Baxter (1995). The response of domestic output is hump-shaped, driven by the endogenous dynamics of capital accumulation and labour supply. Initially, strong wealth effects in anticipation of future income raise demand for domestic goods well in excess of supply, thus causing the terms of trade to appreciate. Over time, as output rises towards its peak, the terms of trade response switches sign relative to the initial equilibrium and depreciate. A value of the trade elasticity as high as 4 turns out to be crucial for these results, as the world economy needs to be able to absorb the future larger supply of domestic tradables—output is hump shaped in equilibrium—with a relatively contained fall in their price. Also crucial is the fact that agents can borrow and lend in the international financial markets: without inter-temporal trade, domestic agents would not be able to raise domestic absorption sharply above supply in the short run, in anticipation of future productivity and income gains, causing the impact appreciation of the terms of trade. Thus, relative to financial autarky, inter-temporal trade allows domestic agents to capture additional benefits from their own productivity gains: the international consumption spillovers from the short run appreciation are in fact negative. For this reason, introducing
international bonds in this economy has negative implications for consumption risk sharing. Because of the high trade elasticity, however, the model yields international prices that are not as volatile as in the data.

A defining feature of our analysis is that strong wealth and demand effects are generated endogenously by productivity shocks following an autoregressive process. However, strong demand movements could result from exogenous shocks to preferences even when markets are complete, as we show in an experiment in the paper, or exogenous shocks to expectations about future income and productivity. Our results indicate that introducing shocks to tastes or expectations is not necessary to reconcile the Backus–Smith puzzle with standard business cycle models. In this respect, our analysis also provides a bridge between general equilibrium models of the international business cycle and traditional models of the Mundell–Fleming vintage. According to the latter, the international business cycle is driven by demand shocks due to policy and/or exogenous changes in expectations. In response to these shocks, a rise in domestic absorption causing an output boom, appreciates the domestic currency in real term, and crowds out net exports. Our results can be interpreted as a quantitative reconsideration of “crowding out effects” in general equilibrium—resulting from the endogenous transmission of productivity disturbances.

The paper is organized as follows. The next section provides a brief summary of the evidence on the correlation between relative consumption and the RER for industrialized countries; Section 3 provides an analytical characterization of the links between these two variables in a simple two-country endowment economy model. Section 4 introduces our full model, whose calibration is presented in Section 5. Section 6 presents and discusses the quantitative predictions of the model, including extensive sensitivity analysis. The final section offers some concluding remarks.

2. INTERNATIONAL CONSUMPTION RISK SHARING AND THE BACKUS–SMITH PUZZLE

In this section, we restate the Backus and Smith (1993) puzzle, looking at the evidence for most OECD countries. As pointed out by Backus and Smith (1993), an internationally efficient allocation implies that the marginal utility of consumption, weighted by the RER, should be equalized across countries:

\[
\frac{P^*_t}{P_t} U_{c,t} = U^*_{c^*,t},
\]  

(1)

where the RER is customarily defined as the ratio of foreign \((P^*_t)\) to domestic \((P_t)\) price level, expressed in the same currency units (via the nominal exchange rate), \(U_{c,t} \) \((U^*_{c^*,t})\) denotes the marginal utility of consumption, and let \(C_t\) and \(C^*_t\) denote domestic and foreign consumptions, respectively. Intuitively, a benevolent social planner would allocate consumption across countries such that the marginal benefits from an extra unit of foreign consumption equal its marginal costs, given by the domestic marginal utility of consumption times the RER \(\frac{P^*_t}{P_t}\), that is, the relative price of \(C^*_t\) in terms of \(C_t\).

If a complete set of state-contingent securities is available, the above condition holds in a decentralized equilibrium independently of trade frictions and goods market imperfections (including shipping and trade costs, as well as sticky prices or wages) that can cause large deviations from the law of one price and PPP.\(^2\) Under the additional assumption that agents have preferences represented by a time-separable, constant-relative-risk-aversion utility function of consumption.

\(^2\) It is only when PPP holds (i.e. RER = 1) that efficient risk sharing implies equalization of the *ex-post* marginal utility of consumption—corresponding to the simple notion that complete markets imply a high cross-country correlation of consumption.
the form \( \frac{C_1 - \sigma}{1 - \sigma} \), with \( \sigma > 0 \), (1) translates into a condition on the correlation between the (logarithm of the) ratio of domestic to foreign consumption and the (logarithm of the) RER. Against the hypothesis of perfect risk sharing, many empirical studies have found this correlation to be significantly below 1, or even negative (in addition to Backus and Smith, 1993, see for instance Kollmann, 1995; and Ravn, 2001).

Table 1 reports the correlation between RER and relative consumption for individual OECD countries relative to the U.S. and to a trade-weighted aggregate of all the other countries, respectively. Since we use annual data, we report the correlations for both the HP-filtered and first-differenced series. As shown in the table, RER and relative consumption are negatively correlated for most OECD countries. The highest correlation is as low as 0.72 (Ireland vis-à-vis other OECD countries), and most correlations are in fact negative—the median of the table entries in the first two columns are \(-0.42\) and \(-0.27\), respectively.

It is useful to note that the evidence on the Backus–Smith puzzle in Table 1 has a counterpart in the correlation between each country’s terms of trade and its relative output. The correlation between these two variables (computed for each country as described in Table 1) tends to be negative or small in all countries but Ireland and the U.K. For the U.S., the correlation is \(-0.33\); the median correlation for either HP-filtered or first-difference data is \(-0.19\) and \(-0.09\), respectively. We will argue below that movements in the terms of trade contribute to our understanding of the Backus–Smith puzzle.

Consistent with other studies, the evidence presented in Table 1 is prima facie at odds with open-economy models assuming a complete set of state-contingent securities. Given that debt and equity trade, the most transparent means of consumption smoothing, are arguably less operative

3. Lewis (1996) rejects non-separability of preferences between consumption and leisure as an empirical explanation of the low correlation of consumption across countries.
4. Statistics for all countries are reported in the web appendix.

\[ C_1 - \sigma \]

\[ \frac{1}{1 - \sigma} \]

\[ \frac{\log \text{RER}}{\log \text{Relative Consumption}} \]

\[ \log \text{Relative Consumption} \]
across borders than within a country, a natural first step to account for the apparent lack of risk sharing is to assume that only a limited number of securities are internationally traded. Restricting the set of assets that agents can use to hedge country-specific risk breaks the tight link between RER and the marginal utility of consumption implied by (1). It should therefore be an essential feature of models trying to account for the stylized facts summarized in Table 1.

Yet, it is now well understood that allowing for incomplete markets may not be enough to bring models in line with the evidence of lack of risk sharing. To start with, in the face of transitory shocks, trade in an international, uncontingent bond may bring the equilibrium allocation quite close to the efficient one. Intuitively, if agents in one country get a positive output shock, they will want to lend to the rest of the world, so that consumption increases both at home and abroad. This result has initially been derived in one-good models, abstracting from movements in relative prices (see, for example, Baxter and Crucini, 1995). However, terms-of-trade movements also impinge on the international transmission of shocks and can even ensure perfect risk sharing independently of trade in financial assets—a point underscored by Cole and Obstfeld (1991) and Corsetti and Pesenti (2001, 2005). Positive productivity shocks in one country that moderately depreciate the domestic terms of trade and the RER will allow consumption abroad to increase to some extent, though less than domestic consumption, thus resulting in a tight positive link between international relative prices and cross-country consumption. More recently, Chari, Kehoe and McGrattan (2002) defines the “Backus–Smith anomaly”, emphasizing that the correlation between relative consumption and the RER remains close to one in incomplete market models assuming trade in one uncontingent international bond, for a large set of model specifications (including sticky prices).

In light of these considerations, the Backus–Smith puzzle provides an important test of open economy models—more specifically, it provides a test of the international transmission mechanism that involves both theoretical and quantitative features.

3. INTO THE PUZZLE: INTERNATIONAL TRANSMISSION OF SUPPLY SHOCK AND RISK SHARING

This section analyses core general equilibrium links between relative consumption, relative output and the RER. In the spirit of Cole and Obstfeld (1991), we carry out our theoretical enquiry using a two-country, two-good endowment economy. Our focus will be on the international transmission of supply shocks, under different assumptions on asset markets. While our full model, shown in Section 4, is much richer than the economy represented here, the theoretical results derived below generalize quite nicely to our full-fledged quantitative analysis. For comparison, we adopt a consistent notation across the two. As in Cole and Obstfeld (1991), we will first focus on the two natural benchmarks of complete markets and financial autarky. Then we will extend our analysis introducing international trade in uncontingent bonds.

3.1. An endowment economy set-up

Consider a two-country, two-good endowment economy. We refer to the two countries as “Home” and “Foreign”, denoted H and F. For the Home representative consumer, consumption is given by the following constant elasticity of substitution (CES) aggregator

\[ C = C_T = \left[ a_H^{-\rho} C_H^\rho + a_F^{-\rho} C_F^\rho \right]^{1/\rho}, \quad \rho < 1, \]  

(2)

where \( C_{H,t} (C_{F,t}) \) is the domestic consumption of Home (Foreign) produced good, \( a_H \) is the share of the domestically produced good in the Home consumption expenditure, \( a_F \) is the...
corresponding share of imported goods, with \( a_F = 1 - a_H \). Note that, since all goods are traded internationally, total consumption coincides with consumption of tradables \( C_T \).

Define \( P_{H,t} \) (\( P_{F,t} \)) as the price of the Home (Foreign) good and \( \tau = \frac{P_F}{P_H} \) the terms of trade, that is, the relative price of Foreign goods in terms of Home goods. Note that an increase in \( \tau \) implies a deterioration of the terms of trade. The welfare-based consumption price index \( P \) (which in our simple economy coincides with the price of tradable consumption \( P_T \)) is

\[
P = P_T = \left[ a_H P_H^{\rho/(\rho-1)} + (1 - a_H) P_F^{\rho/(\rho-1)} \right]^{(\rho-1)/\rho}.
\]

Domestic demand for Home goods can be written as

\[
C_H = a_H \left( \frac{P_H}{P} \right)^{-\omega} C,
\]

where the demand’s price elasticity coincides with the elasticity of substitution across the two goods, \( \omega = (1 - \rho)^{-1} \). Analogous expressions can be derived for the Foreign country—below foreign variables are denoted with a star \( \"\ast\" \).

As in the previous section, we assume identical power utility. Letting \( Y_H \) denote Home (tradable) output and \( Y_F \) Foreign output, the resource constraint for both domestic and foreign tradables is \( Y_H = C_H + C_H^* \), and \( Y_F = C_F + C_F^* \).

### 3.2. Positive terms of trade spillovers under complete markets

With complete markets, perfect consumption insurance optimally insulates relative wealth from price movements. This shapes the equilibrium response of relative prices to output shocks. Specifically, any gain in domestic output is necessarily associated with a terms of trade depreciation: international consumption spillovers of productivity shocks are unambiguously positive.

To see this, use the resource constraints of the economy together with the efficient risk-sharing condition (1) to obtain

\[
\frac{Y_H}{Y_F} = \frac{a_H \left( \frac{P_H}{P} \right)^{-\omega} C + a_H^* \left( \frac{P_H}{P} \right)^{-\omega} C \left( \frac{P_F^*}{P} \right)^{-\sigma^{-1}}}{(1 - a_H) \left( \frac{P_F}{P} \right)^{-\omega} C + (1 - a_H^*) \left( \frac{P_F}{P} \right)^{-\omega} C \left( \frac{P_F^*}{P} \right)^{-\sigma^{-1}}} = \tau^{\omega a_H + a_H^* \left[ \frac{a_H^* + (1 - a_H) \tau^{1-a_H}}{a_H^* + (1 - a_H^*) \tau^{1-a_H}} \right]^{(\omega - \sigma^{-1})/(1 - \omega)}}
\]

By taking a log-linear approximation of the above equation around a symmetric equilibrium (with \( a_H = 1 - a_H^* \) and \( Y_H = Y_F^* \)), the link between relative output (endowment) changes, and the terms of trade/RER can be expressed as follows:

\[
\hat{\tau} = \frac{\sigma}{[1 - (2a_H - 1)^2] \omega \sigma + (2a_H - 1)^2 (\hat{Y}_H - \hat{Y}_F)}.
\]

Since \( 0 \leq a_H \leq 1 \), the coefficient above is positive: with efficient risk sharing, an increase in domestic output necessarily leads to a fall in the Home terms of trade, benefiting Foreign consumers.
3.3. Wealth effects and the international transmission mechanism under financial autarky

When markets are incomplete, the interplay of substitution and wealth effects leads to a much richer array of results relative to the case of perfect risk insurance. At the core of our results is the notion that strong wealth effects can drive aggregate demand for domestic goods above supply, driving up their prices in spite of positive endowment shocks. In this subsection we analyse this possibility assuming financial autarky.

3.3.1. Terms of trade and the world demand for domestic goods. To introduce our analysis it is useful to reconsider the analytics of wealth effects and demand in our general equilibrium model. Since under financial autarky consumption expenditure has to equal current income in each period, that is, $\frac{PC}{P_H} = Y_H$, domestic demand for Home goods can be written as

$$C_H = \frac{a_H}{a_H + (1-a_H)\tau^{1-\omega}} Y_H.$$  

Taking the derivative of $C_H$ with respect to $\tau$, we can decompose the response of the domestic demand for the Home good to a fall in its price (corresponding to an increase in $\tau$) into a substitution effect (SE), and an income effect (IE):\(^5\)

$$\frac{\partial C_H}{\partial \tau} = \omega \frac{a_H(1-a_H)\tau^{-\omega}}{[a_H + (1-a_H)\tau^{1-\omega}]^2} Y_H - \frac{a_H(1-a_H)\tau^{1-\omega}}{[a_H + (1-a_H)\tau^{1-\omega}]^2} Y_H > 0 \iff \omega > 1. \quad (5)$$

Note that, in general equilibrium, what drives the negative IE from lowering the Home good price is the fall in the value of the Home endowment $Y_H$ in the world economy, changing relative wealth. From the above expression, it is apparent that, in equilibrium, the Home demand for the Home good $C_H$ can either increase or decrease in response to a deterioration of the Home terms of trade $\tau$, depending on the relative strength of the two effects above. Specifically, when $\omega > 1$, the positive SE from lower prices is larger in absolute value than the negative IE. A fall in the relative price of the domestic tradable—an increase in $\tau$—will unambiguously raise its domestic demand. The opposite is true when $\omega < 1$: the negative IE will more than offset the SE. Thus, with a low trade elasticity, a terms-of-trade depreciation will reduce the domestic demand for the Home tradable.

On the contrary, Foreign consumption of Home tradables, $C^*_F$, will unambiguously increase with $\tau$, independently of $\omega$. This is because, in equilibrium, a rise in $\tau$ improves the Foreign terms of trade, raising the value of Foreign output and income. Income and substitution effects move Foreign consumption in the same direction.\(^6\)

5. By a straightforward derivation of the Slutsky equation, the SE is obtained from the compensated demand function $x_H$:

$$\frac{\partial x_H}{\partial \tau} = \omega \frac{a_H(1-a_H)\tau^{-\omega}}{[a_H + (1-a_H)\tau^{1-\omega}]^2} Y_H.$$  

In our analysis below, the IE we refer to is a general-equilibrium effect, which depends, among other things, on the structure of financial markets. To wit: it is easy to show that, in the complete market setting of the Section 3.2, the SE can never be below the IE (regardless of the value of $\omega$). As explained in the text, results are sharply different in the incomplete-market version of our economy.

6. Using self-explanatory notation

$$\frac{\partial C^*_H}{\partial \tau} = \omega \frac{a_H^*(1-a_H^*)\tau^{-\omega}}{[(1-a_H^*)\tau^{1-\omega} + a_H^*Y^*_F]^2} Y^*_H + \frac{a_H^*(1-a_H^*)\tau^{1-\omega}}{[(1-a_H^*)\tau^{1-\omega} + a_H^*Y^*_F]^2} Y^*_F > 0.$$
These results together allow us to characterize the equilibrium properties of the world demand for Home goods $C_H + C_H^*$ as a function of $\omega$. As long as the negative IE in the Home country are not too strong, the world demand for Home goods will fall with an increase in their price. But if the trade elasticity $\omega$ is sufficiently below 1 (combined with some degree of Home bias in consumption, that is, a large $a_H$ relative to $a_H^*$), it is possible that the equilibrium response of the world demand be a fall. This happens when strong negative IE driving $C_H$ dominate any positive SE worldwide, as well as positive IE abroad.\(^7\)

This characterization of the world demand for domestic output in turn provides the analytical underpinning of the general equilibrium effects of endowment shocks. If $\omega$ is sufficiently high, a positive shock to Home output $Y_H$ is matched by a higher world demand at lower prices: the Home terms of trade depreciate. Since Foreign households raise their consumption of Home tradables taking advantage of better import prices, the international transmission is positive. If, instead, $\omega$ is low enough, a positive supply shock to $Y_H$ cannot be matched by an increase in world demand at lower prices: the Home terms of trade need to appreciate. The international transmission in this case is negative: a positive domestic supply shock has a negative impact on consumption and welfare abroad.

Formally, take a log-linear approximation of the market clearing condition for Home tradables around a symmetric equilibrium so as to obtain

$$\hat{\tau} = \frac{Y_H - Y_F^*}{1 - 2a_H(1 - \omega)},$$

$$\text{RER} = \frac{2a_H - 1}{1 - 2a_H(1 - \omega)}(Y_H - Y_F^*).$$

(6) (7)

For given movements in relative output, the sign of the response of international relative prices depend on whether the elasticity of substitution $\omega$ is above or below the threshold

$$\tilde{\omega} = \frac{2a_H - 1}{2a_H}.$$

When $\omega > \tilde{\omega}$, both the RER and the terms of trade depreciate in response to a positive Home supply shock; in equilibrium the world demand schedule is falling in prices, and the international transmission is positive. Otherwise, both prices appreciate, and the international transmission is negative. Observe that, under our assumption of home bias in consumption ($a_H > 1/2$), $\tilde{\omega} < 1/2$.

The expressions (6) and (7) also unveil that the volatility of the terms of trade and the RER in response to output shock is not monotonic in the elasticity of substitution: it is falling in $\omega$ in the region where $\omega > \tilde{\omega}$, increasing in $\omega$ otherwise. Furthermore, international price volatility is very high for $\omega$ close to the cut-off point, whereas the slope of the demand function becomes steeper and steeper before changing sign. An important implication is that there will be two values of $\omega$ (below and above the cut-off point) that yield the same volatility of the terms of trade and the RER in response to productivity shocks.

3.3.2. Exchange rates and consumption. The transmission mechanism discussed above is crucial to understand risk sharing and the equilibrium comovements between the RER and relative consumption. With incomplete markets the scope for insurance against country-specific shocks is limited and swings in international relative prices—driving the valuation of national output apart—expose consumers to potentially strong relative wealth shocks.

\(^7\) We are grateful to Fabrizio Perri for suggesting this line of exposition. In our simplified model analyzed in this section, strong IE raise the possibility of multiple steady states (e.g. see the discussion in Corsetti and Dedola, 2002). It is worth stressing, however, that the specification of preferences in the model we use in our numerical exercises below always ensures a unique steady state. A formal proof is available from the authors.
With financial autarky, the balanced-trade condition can be manipulated to derive an expression for relative consumption as a function of the terms of trade:

$$\tau C_F = C_H^* \iff C C^* = \frac{a_H}{1-a_H} \tau^\omega - 1 \left[ \frac{a_H^* + (1-a_H^*) \tau^{1-\omega}}{a_H + (1-a_H) \tau^{1-\omega}} \right]^{\omega/(1-\omega)}; \quad (8)$$

from this, we can then derive the following log-linearized relationship between the RER and relative consumption:

$$\hat{\text{RER}} = \frac{2a_H - 1}{2a_H\omega - 1} (\hat{C} - \hat{C}^*). \quad (9)$$

The relation between RER and relative consumption can have either sign, once again depending on the values of $\omega$ and $a_H$. Specifically, with home bias in consumption, it will be negative when $\omega < \frac{1}{2a_H} < 1$.

Note that the threshold $1/2a_H$ is larger than $\bar{\omega}$, the cut-off point at which the terms of trade response to output shock changes sign. It follows that a negative Backus–Smith correlation can correspond to different patterns of the international transmission. For a low enough elasticity, a Home supply shock causes an appreciation of the Home terms of trade and the RER, driving Home consumption above Foreign consumption.\(^8\)

Contrast (9) with the condition for efficient risk sharing (1), which in our economy takes the following log-linear form

$$\hat{\text{RER}} = \sigma (\hat{C} - \hat{C}^*).$$

The two expressions are identical if $\omega = 2\frac{1-\sigma}{2a_H\sigma}$. Using (4) and (7), it is easy to show that, for parameters satisfying this condition, the response of relative consumption to endowment shocks will be the same across the two economies (up to a first-order approximation). A special case is $\omega = 1$ and no home bias in consumption ($a_H = a_H^* = 1/2$): regardless of the structure of asset markets, and for any $\sigma > 1$, consumption will be equalized across countries.\(^9\) This is the benchmark economy analysed by Cole and Obstfeld (1991), where equilibrium relative price movements exactly offset changes in output quantities, leaving cross-country relative wealth and consumption unchanged. Even under financial autarky, agents can achieve the optimal degree of international insurance.

### 3.4. Wealth effects and the international transmission mechanism with international trade in bonds

In this subsection, we extend our theoretical enquiry to the case of an economy with international trade in bonds, allowing agents to smooth consumption through cross-border borrowing and lending. In a bond economy, relative to our previous findings, the negative transmission mechanism by which supply shocks can cause temporary appreciation of the terms of trade, and induce a negative Backus–Smith correlation, can take place also for an elasticity of substitution sufficiently larger than 1. The main idea here is that domestic wealth rises strongly in the short run, in anticipation of future output gains.

For the sake of clarity, we derive an analytical expression (up to a first-order approximation) relating terms of trade to output shocks at different horizons, focusing on the special case of log

\(^8\) When $\frac{2\omega - 1}{2a_H} < \omega < \frac{1}{2a_H}$, the Home terms of trade and exchange rate depreciate in response to a Home supply shock, to such an extent that relative domestic wealth decreases, driving foreign consumption above domestic consumption (without necessarily implying “immiserizing growth”). However, this case is not supported by our calibration strategy based on the method of moments (see Sections 5 and 6 below).

\(^9\) In this case the equivalence result holds also in the fully non-linear equilibrium.
utility, and letting the rate of time preferences go to 0. Also, we assume that shocks increase the endowment over time, reaching a permanently higher level, that is, \( 0 < \hat{Y}_{H,t} \leq \hat{Y}_H \). Under these simplifying assumptions, we obtain

\[
\hat{\tau}_t = \frac{(\hat{Y}_{H,t} - \hat{Y}_H) - (\hat{Y}_{F,t} - \hat{Y}_F)}{1 - 4a_H(1 - a_H)(1 - \omega)} + \frac{\hat{Y}_H - \hat{Y}_F}{1 - 2a_H(1 - \omega)}. \tag{10}
\]

Here, \( \hat{Y}_H (\hat{Y}_F) \) denotes the percentage deviation of Home (Foreign) output from the initial steady state in the long run. The first term in the above expression is unambiguously negative; with a trade elasticity larger than 1, the second term unambiguously positive.\(^{10}\) Intuitively, with \( \omega > 1 \) the deterioration of the terms of trade in the long run is less than proportional to the change in endowment. Hence in the long run the value of the Home output rises relative to world output. This translates into a positive wealth effect generating a short-run domestic consumption boom. Because of home bias, domestic consumption falls disproportionately on domestic goods, whose supply rises less in the short run than in the long run. Unless the short-run gains in output are already large (\( i.e. \) unless \( \hat{Y}_H \) is close to \( \hat{Y}_H \)), the domestic consumption boom creates excess demand for the Home goods, triggering an impact appreciation of the terms of trade. Over time, as the dynamic of Home output endowment fills the gap with demand, the terms of trade appreciation switches to a depreciation (also relative to the initial equilibrium). Thus, endowment shocks with the features just specified can induce a dynamic response of the terms of trade, such that short-run appreciation is followed by depreciation in the long run. With home bias in consumption, a short-run terms-of-trade appreciation will obtain with both a large enough gap between short-run and long-run output (\( \hat{Y}_H - \hat{Y}_H \) be sufficiently negative) and a trade elasticity sufficiently larger than 1.\(^{11}\)

Under the simplifying assumptions stated above, we can write the relation between relative consumption and the RER distinguishing a short-run component from a long-run one:

\[
(\hat{C}_t - \hat{C}_t^*) = \hat{\text{RER}}_t + \frac{2a_H(\omega - 1)}{2a_H - 1} \hat{\text{RER}}.
\]

Endowment shocks that generate a domestic consumption boom and appreciate the Home terms of trade in the short run induce a negative correlation between relative consumption and the RER (recall that the \( \hat{\text{RER}}_t \), and the terms of trade move in the same direction). However, this negative correlation will never be perfect, as a trade elasticity larger than 1 implies that the long-run RER unambiguously depreciates (the second term on the R.H.S. of the above expression is unambiguously positive).

In our endowment economy with trade in bonds, output gains that are raising over time, and a trade elasticity sufficiently larger than 1 account for the Backus–Smith puzzle. An important question for the international business cycle literature we address below is whether a full-fledged quantitative model can address the Backus–Smith anomaly, when productivity shocks follow an autoregressive stationary process. A specific and novel contribution of this paper is to show that

10. Observe that in the case of permanent shocks with no dynamics, that is, the endowment jumps to the new long-run level immediately so that \( \hat{Y}_{H,t} = \hat{Y}_H \), agents have no incentive to smooth consumption. It is apparent that the response of the terms of trade is the same as under financial autarky, as (10) coincides with (6). The simplifying assumptions underlying the decomposition in (10), however, rule out terms of trade appreciation in response to temporary shocks—a result that is instead possible in more general models as shown in the rest of the paper. Precisely, Section 6.1 provides a detailed analysis of temporary shocks in our full-fledged economy with capital accumulation and bond trade, generalizing our analytical results in the text above.

11. Specifically, in our example the following conditions must be satisfied:

\[
\hat{Y}_H > \frac{\hat{Y}_H}{2a_H - 1} > 0 \quad \text{and} \quad \omega > \frac{(2a_H - 1)(\hat{Y}_H - \hat{Y}_F) + [1 - 4a_H(1 - a_H)]\hat{Y}_H}{2a_H(2a_H - 1)(\hat{Y}_H - \hat{Y}_F) - 2(1 - a_H)\hat{Y}_H} > 1.
\]
this is indeed the case, provided the persistence of the autoregressive process of the shocks is high enough, and the trade elasticity is set to the high values suggested by the trade literature. As explained below, in this case the endogenous dynamics of international trade in bonds, capital accumulation, and labour supply is such that the output process displays a hump-shaped pattern, even though productivity gains (gently) fall over time after the initial shock.12

4. THE MODEL

In this and the next section we develop our model, which will then be solved by employing standard numerical techniques. Our world economy consists of two countries of equal size, as before denoted H and F, each specialized in the production of an intermediate, perfectly tradable good. In addition, each country produces a non-tradable good. This good is either consumed or used to make intermediate tradable goods H and F available to domestic consumers. In what follows, we describe our set-up focusing on the Home country, with the understanding that similar expressions also characterize the Foreign economy—as above, starred variables refer to Foreign firms and households.

4.1. The firms’ problem

Firms producing Home tradables (H) and Home non-tradables (N) are perfectly competitive and employ a technology that combines domestic labour and capital inputs, according to the following Cobb–Douglas functions:

\[ Y_H = Z_H K_H^{1-\xi} L_H^\xi \]
\[ Y_N = Z_N K_N^{1-\zeta} L_N^\zeta, \]

where \( Z_H \) and \( Z_N \) are exogenous random disturbances following a statistical process to be determined below. We assume that capital and labour are freely mobile across sectors. The problem of these firms is standard: they hire labour and capital from households to maximize their profits:

\[ \pi_H = \overline{P}_{H,t} Y_{H,t} - W_t L_{H,t} - R_t K_{H,t} \]
\[ \pi_N = P_{N,t} Y_{N,t} - W_t L_{N,t} - R_t K_{N,t}, \]

where \( \overline{P}_{H,t} \) is the wholesale price of the Home-traded good, and \( P_{N,t} \) is the price of the non-traded good. \( W_t \) denotes the wage rate, while \( R_t \) represents the capital rental rate.

Firms in the distribution sector are also perfectly competitive. They buy tradable goods and distribute them to consumers using non-traded goods as the only input in production. As in Burstein, Neves and Rebelo (2003) and Corsetti and Dedola (2005), we assume that bringing one unit of traded goods to Home (Foreign) consumers requires \( \eta \) units of the Home (Foreign) non-traded goods. Although this assumption is stark, a low elasticity of substitution between traded goods and non-traded distribution services is quite reasonable—in line with the conventional wisdom in industrial organization, as synthesized by Tirole’s factual remark that “production and retailing are complements, and consumers often consume them in fixed proportions” (Tirole, 1995, p. 175). Overall, distributive trade accounts for an important share of the retail price of consumption goods: for the U.S., including wholesale and retail services, marketing, advertising, and local transportation, the average distribution margin is as high as 50% (see Burstein, Neves and Rebelo, 2003; and Anderson and van Wincoop, 2004).

12. In related work (Corsetti, Dedola and Leduc, 2006, 2007), we provide evidence on the dynamics of the terms of trade and the international transmission following productivity shocks in the manufacturing sector, based on VAR models for a sample of industrial countries. In the case of the U.S., we find evidence of terms of trade appreciation.
4.2. The households’ problem

4.2.1. Preferences. The representative Home agent in the model maximizes the expected value of her lifetime utility, given by

$$E \left\{ \sum_{t=0}^{\infty} U[C_t, \ell_t] \exp \left[ -\sum_{\tau=0}^{t-1} -\nu(C_{\tau}, \ell_{\tau}) \right] \right\},$$  \hspace{1cm} (11)

where instantaneous utility $U$ is a function of a consumption index, $C$, and leisure, $(1-\ell)$. Following Schmitt-Grohé and Uribe (2003), we assume an endogenous discount factor $\nu(C_{t}, \ell_{t})$, which is a function of the average per capita level of consumption, $C_t$, and hours worked, $\ell_t$. Foreign agents’ preferences are symmetrically defined. It can be shown that, for all parameter values used in the quantitative analysis below, these preferences guarantee the presence of a locally unique symmetric steady state, independent of initial conditions.  

The full consumption basket, $C_t$, in each country is defined by the following CES aggregator of consumption of both tradable and non-tradable goods

$$C_t \equiv \left[ a_T \left( 1-\phi \right) C_T, t^\phi + a_N \left( 1-\phi \right) C_N, t^\phi \right]^{1/\phi}, \quad \phi < 1, $$  \hspace{1cm} (12)

where $a_T$ and $a_N$ are the weights on the consumption of traded and non-traded goods, respectively, and $\frac{1}{1-\phi}$ is the CES between $C_N, t$ and $C_T, t$. As in Section 3, the consumption index of traded goods $C_T, t$ is given by (2) including $C_H, t$ (the consumption of the Home-traded goods) and $C_F, t$ (consumption of Foreign-traded goods).

4.2.2. Price indexes and RER. A notable feature of our specification is that, because of distribution costs, there is a wedge between the producer price and the consumer price of each good. Let $P^*_H, t$ and $P^*_C, t$ denote the price of the Home-traded good at the producer and consumer level, respectively. Let $P_N, t$ denote the price of the non-traded good that is necessary to distribute the tradable one. With competitive firms in the distribution sector, the consumer price of the tradable good is simply

$$P^*_{H, t} = \eta P^*_{N, t}. $$  \hspace{1cm} (13)

We hereafter write the utility-based CPIs, whereas the price index of tradables $P_T$ is given by (3):

$$P_t = \left[ a_T P_{t, T}^{\phi/(\phi-1)} + a_N P_{t, N}^{\phi/(\phi-1)} \right]^{(\phi-1)/\phi}. $$  \hspace{1cm} (14)

Foreign prices, denoted with an asterisk and expressed in the same currency as Home prices, are similarly defined. Observe that the law of one price holds at the wholesale level but not at the consumer level, so that $P^*_{H, t} = P_{H, t}^*$ but $P_{H, t} \neq P^*_{H, t}$. Our model is thus in the tradition of the international macroeconomics literature pointing to distribution services as a key reason for the failure of PPP—in addition to the presence of non-tradable goods in the consumption basket and home bias in the domestic demand for tradables. Dornbusch (1989), for instance, suggests that these services may provide an explanation for his finding that the price of an identical consumption basket is higher in high-income economies than in low-income ones. This tradition has been recently revived by a growing empirical and quantitative literature—see Burstein, Eichenbaum and Rebelo (2005), Corsetti and Dedola (2005), and Crucini, Telmer and Zachariadis (2005) among others—emphasizing the role of distribution in explaining RER.

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13. A unique invariant distribution of wealth under these preferences will allow us to use standard numerical techniques to solve the model around a stable nonstochastic steady state when only a non-contingent bond is traded internationally (see Obstfeld, 1990; Mendoza, 1991; and Schmitt-Grohé and Uribe, 2003).
movements. The findings by this literature provide a strong motivation for modelling the distribution sector in a quantitative analysis of the Backus–Smith puzzle. As shown below, the inclusion of distribution substantially improves the match between the model and essential stylized facts regarding RER. A good match with the evidence in this dimension is arguably an essential prerequisite of quantitative models whose aim is to generate a low, negative correlation between relative consumption and the RER—making sure that such result is tied in with realistic predictions about the behaviour of the latter.

4.2.3. Budget constraints and asset markets. Home and Foreign agents hold an international bond, $B_{H,t}$, which pays in units of Home aggregate consumption and is zero in net supply. Agents derive income from working, $W_{t}t_{t}$, from renting capital to firms, $R_{t}K_{t}$, and from interest payments, $(1 + r_{t})B_{H,t}$, where $r_{t}$ is the real bond’s yield, paid at the beginning of period $t$ but known at time $t - 1$. The individual flow budget constraint for the representative agent in the Home country is therefore

$$
P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_{N,t}C_{N,t} + B_{H,t+1} + \overline{P}_{H,t}I_{H,t} \leq W_{t}t_{t} + R_{t}K_{t} + (1 + r_{t})B_{H,t}.
$$

We assume that investment is carried out in Home-tradable goods. The law of motion for the aggregate capital stock is given by

$$
K_{t+1} = I_{H,t} + (1 - \delta)K_{t}.
$$

As opposed to consumption goods, we assume that investment goods do not require distribution services. The price of investment is therefore equal to the wholesale price of the domestic-traded good, $\overline{P}_{H,t}$. Finally, we assume that the capital stock, $K$, can be freely reallocated between the traded ($K_{H}$) and non-traded ($K_{N}$) sectors.

The household’s problem then consists of maximizing lifetime utility, defined by (11), subject to the constraints (15) and (16).

4.3. Competitive equilibrium

Let $s_{t} = [B_{H}, K, \kappa^{*}; Z]$ denote the state of the world at time $t$, where $Z = [Z_{H}, Z_{F}, Z_{N}, Z_{N}^{*}]$. A competitive equilibrium is a set of Home agent’s decision rules $C_{H}(s)$, $C_{F}(s)$, $C_{N}(s)$, $I_{H}(s)$, $I_{F}(s)$, $B_{H,t+1}(s)$; a set of Foreign agent’s decision rules $C_{H}^{*}(s)$, $C_{F}^{*}(s)$, $C_{N}^{*}(s)$, $I_{H}^{*}(s)$, $I_{F}^{*}(s)$, $B_{H,t+1}^{*}(s)$; a set of Home firms’ decision rules $K^{*}(s)$, $K_{N}^{*}(s)$, $L_{H}^{*}(s)$, $L_{N}^{*}(s)$; a set of Foreign firms’ decision rules $K_{H}^{*}(s)$, $K_{N}^{*}(s)$, $L_{H}^{*}(s)$, $L_{N}^{*}(s)$; a set of pricing functions $P_{H}(s)$, $P_{F}(s)$, $\overline{P}_{H}(s)$, $\overline{P}_{F}(s)$, $P_{N}(s)$, $P_{N}^{*}(s)$, $W(s)$, $W^{*}(s)$, $R(s)$, $R^{*}(s)$, $r(s)$ such that (i) the agents’ decision rules solve the households’ problems; (ii) the firms’ decision rules solve the firms’ problems; and (iii) the appropriate market-clearing conditions hold: $l = L_{H} + L_{N}$; $K = K_{H} + K_{N}$; $Y_{N} = C_{N} + \eta C_{H} + \eta C_{F}$; $Y_{T}^{*} = I_{H}^{*} + C_{H}^{*} + C_{F}^{*}$; $I^{*} = L_{F}^{*} + L_{N}^{*}$; $K^{*} = K_{H}^{*} + K_{N}^{*}$; $Y^{*}_{N} = C_{N}^{*} + \eta^{*}C_{H}^{*} + \eta^{*}C_{F}^{*}$; $Y^{*}_{T} = I_{F} + C_{F} + C_{F}^{*}$; and $B_{H,t+1} + B_{H,t}^{*} = 0$.

In the remainder of the paper, the price of Home aggregate consumption $P_{t}$ will be taken as the numeraire. Hence, the RER will be given by the price of Foreign aggregate consumption $P_{t}^{*}$ in terms of $P_{t}$.

14. $B_{H,t}$ denotes the Home agent’s bonds accumulated during period $t - 1$ and carried over into period $t$.

15. We also conduct sensitivity analysis on our specification of the investment process, see Section 6.2 below.
5. MODEL CALIBRATION

Our baseline calibration is shown in Table 2, with the exception of the elasticity of substitution between Home and Foreign tradables. This is because we calibrate this parameter using two different strategies, described at the end of this section. Most parameters’ values reported in Table 2 are similar to those adopted by Stockman and Tesar (1995), who calibrate their models to the U.S. relative to a set of OECD countries on annual data. Note that we assume symmetry across countries.

Preferences and production. Consider first the preference parameters. Assuming a utility function of the form

\[ U[C_t, \ell_t] = \left[ \frac{C_t^\alpha (1 - \ell_t)^{1-\alpha}}{1-\sigma} \right]^{\frac{1-\sigma}{1-\sigma}}, \quad 0 < \alpha < 1, \quad \sigma > 0, \]  

(17)

where we set \( \alpha \) so that in steady state, one-third of the time endowment is spent working; \( \sigma \) (risk aversion) is set equal to 2. We assume that the endogenous discount factor has the following form:

\[ \nu(C_t, \ell_t) = \ln(1 + \psi[C_t^\alpha (1 - \ell_t)^{1-\alpha}]), \]

whereas \( \psi \) is chosen such that the steady-state real interest rate is 4% per annum, equal to 0.08. This parameter also determines the speed of convergence to the unique non-stochastic steady state.

The value of \( \phi \) is selected based on the available estimates for the elasticity of substitution between traded and non-traded goods. We use the estimate by Mendoza (1991) referred

| TABLE 2 |
| Parameter values |

Baseline model
Preferences and technology
Risk aversion \( \sigma = 2 \)
Consumption share \( \alpha = 0.34 \)
Elasticity of substitution
Home and Foreign-traded goods \( \rho \) = 0.85
Traded and non-traded goods \( \phi \) = 0.74
Share of Home-traded goods \( a_H \) = 0.72
Share of non-traded goods \( a_N \) = 0.45
Elasticity of the discount factor with respect to \( C \) and \( L \) \( \psi = 0.08 \)
Distribution margin \( \eta = 1.09 \)
Labour share in tradables \( \zeta = 0.61 \)
Labour share in non-tradables \( \xi = 0.56 \)
Depreciation rate \( \delta = 0.10 \)

Productivity shocks
\[ \lambda = \begin{bmatrix} 0.82 & -0.06 & 0.10 & 0.24 \\ -0.06 & 0.82 & 0.24 & 0.10 \\ -0.02 & 0.02 & 0.96 & 0.01 \\ 0.02 & -0.02 & 0.01 & 0.96 \end{bmatrix} \]

Variance–covariance matrix (in per cent)
\[ V(\omega) = \begin{bmatrix} 0.047 & 0.022 & 0.009 & 0.004 \\ 0.022 & 0.047 & 0.004 & 0.009 \\ 0.009 & 0.004 & 0.009 & -0.001 \\ 0.004 & 0.009 & -0.001 & 0.009 \end{bmatrix} \]
to a sample of industrialized countries and set that elasticity equal to 0.74. Stockman and Tesar (1995) estimate a lower elasticity (0.44), but their sample includes both developed and developing countries.

As regarding the weights of domestic and foreign tradables in the tradables consumption basket \((C_T)\), \(a_H\) and \(a_F\) (normalized to \(a_H + a_F = 1\)) are chosen such that imports are 5% of aggregate output in steady state. This corresponds to the average ratio of U.S. imports from Europe, Canada, and Japan to U.S. GDP between 1960 and 2002. The weights of traded and non-traded goods, \(a_T\) and \(a_N\), are chosen as to match the share of non-tradables in the U.S. consumption basket. Over the period 1967–2002, this share is equal to 53% on average. Consistently, Stockman and Tesar (1995) suggest that the share of non-tradables in the consumption basket of the seven largest OECD countries is roughly 50%.

We calibrate \(\xi\) and \(\zeta\), the labour shares in the production of tradables and non-tradables, based on the work of Stockman and Tesar (1995). They calculate these shares to be equal to 61% and 56% , respectively. Finally, we set the depreciation rate of capital equal to 10% annually.

**Productivity shocks.** We previously defined the exogenous state vector as \(Z \equiv \{Z_H, Z_F, Z_N, Z^*_N\}'\). We assume that disturbances to technology follow a trend-stationary AR(1) process

\[
Z' = \lambda Z + u,
\]

whereas \(u \equiv (u_H, u_F, u_N, u^*_N)\) has variance–covariance matrix \(V(u)\), and \(\lambda\) is a 4 x 4 matrix of coefficients describing the autocorrelation properties of the shocks. Since we assume a symmetric economic structure across countries, we also impose cross-country symmetry on the autocorrelation and variance–covariance matrices of the above sectoral process.

Consistent with our model and other open-economy studies (e.g. Backus, Kehoe and Kydland, 1995), we identify technology shocks with Solow residuals in each sector, using annual data in manufacturing and services from the OECD STAN database. Since hours are not available for most other OECD countries, we use sectoral data on employment. An appendix describes our data in more detail.

The bottom panel of Table 2 reports our estimates of the parameters describing the process driving productivity. As found by previous studies, our estimated technology shocks are fairly persistent. On the other hand, in line with empirical studies, we find that spillovers across countries and sectors are not negligible.

**Distribution costs and the price elasticity of tradables.** The introduction of a distribution sector in our model is a novel feature relative to standard business cycle models in the literature. We have already discussed its importance in accounting for deviations from the law of one price at retail level. Before delving into numerical analysis, we need to discuss an important implication of specifying a low degree of substitutability between distribution services and goods, for the price elasticity of tradables. From the representative consumer’s first-order conditions (regardless of frictions in the asset and goods markets), optimality requires that the relative price of the imported good in terms of the domestic tradable at consumer level be equal to the ratio of marginal utilities:

\[
\frac{P_{F,t}}{P_{H,t}} = \frac{\bar{P}_{F,t} + \eta P_{N,t}}{\bar{P}_{H,t} + \eta P_{N,t}} = \frac{1 - a_H}{a_H} \left( \frac{C_{H,t}}{C_{F,t}} \right)^{1/\omega},
\]

where \(\omega = (1 - \rho)^{-1}\) is equal to the elasticity of substitution between Home and Foreign tradables in the consumption aggregator \(C_{T,t}\) and thus to the consumer price elasticity of these goods. Note that \(C_{H,t}/C_{F,t}\) is the inverse of the ratio of real imports to non-exported tradable output net of investment. In analogy to the literature, we can refer to this ratio as the (tradable) import ratio.
Because of distribution costs, the relative price of imports in terms of Home exports at the consumer level does not coincide with the terms of trade $\frac{P_{F,t}}{P_{H,t}}$—as in most standard models (e.g. Lucas, 1982). Let $\mu$ denote the size of the distribution margin in steady state, that is, $\mu = \eta \frac{P_N}{P_H}$. By log linearizing (19), we get

$$\hat{\tau}_t = \frac{1}{\omega(1 - \mu)} \left( \hat{C}_{H,t} - \hat{C}_{F,t} \right)$$

(20)

where the terms of trade $\tau$ is measured at the producer-price level so that $\omega(1 - \mu)$ can be thought of as the producer price elasticity of tradables. Clearly, both $\omega$ and $\mu$ impinge on the magnitude of the international transmission of country-specific shocks through the equilibrium changes in the terms of trade.16

There is considerable uncertainty regarding trade price elasticities. Using aggregate time series data, empirical researchers have found estimates that range from about 0.1 to 2 (see the comprehensive study on G-7 countries by Hooper, Johnson and Marquez, 2000). For instance, for the U.S. Taylor (1993) estimates a value of 0.39, while Whalley (1985) finds it to be 1.5. Correspondingly, there are differences in the quantitative literature. For instance, in a model with traded and non-traded goods similar to ours, Stockman and Tesar (1995) set the parameter $\omega$—directly related to the price elasticity with no distribution costs—equal to 1. Following Whalley (1985), in a model with only tradable goods Backus, Kehoe and Kydland (1995) set it equal to 1.5, whereas Heathcote and Perri (2002) estimate it as low as 0.9. However, these authors also report sensitivity analysis suggesting that much lower values, in the range of 0.5, can improve their model performance in accounting for features of the international business cycle like the volatility of the terms of trade.

Given the uncertainty surrounding the appropriate parameter value, and the key role this elasticity plays in open economy models, we pursue the following calibration approach. First, we set $\mu$ based on estimates of distribution costs. Specifically, according to the evidence for the U.S. economy in Burstein et al. (2003), the share of the retail price of traded goods accounted for by local distribution services ranges between 40% and 50%, depending on the industrial sector. Correspondingly, in their exhaustive survey on trade costs, Anderson and van Wincoop (2004) report that representative local distribution costs account for over 55% of retail prices in industrialized countries. Following the calibration in Burstein, Neves and Rebelo (2003), we set distribution costs to 50%. Second, we calibrate the elasticity of substitution $\omega$ based on the two strategies discussed below.

**Calibrating the trade elasticity.** The first strategy, motivated by our analytical results of Section 3.3, consists in setting the parameter $\omega$ to minimize the distance, between model and data, of the following (equally weighed) four moments: the volatility of the RER, the volatility of the terms of trade, the correlation between the real exchange and the output ratio and the correlation between the RER and net exports.17 By matching the first two moments, we address the

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16. Note that under financial autarky the counterpart of condition (5 ) in our fully specified model with distribution services is:

$$\frac{\partial C_H}{\partial \tau} > 0 \iff \omega(1 - \mu)(1 - a_H) \left( \frac{P_F}{P_H} \right)^{1-\omega} \frac{1-\omega}{\omega} - (1-a_H) \left( \frac{P_F}{P_H} \right)^{1-\omega} a_H \mu > 0.$$  

A positive distribution margin $\mu$ reduces the SE from a deterioration in the terms of trade, while making the IE less negative. The presence of distributive trade causes the consumer price to fall less than one-to-one relative to the relative price of domestic tradables.

17. As is well known, generalized method of moments estimates are unbiased and consistent when moments are weighted equally. Nonetheless, we verified that our estimate of $\omega$ is unchanged when we use the optimal weighting matrix (based on Lee and Ingram, 1991).
following issue: if we choose $\omega$ such that our model gets close to the volatility of the RER and the terms of trade, do these relative price movements amplify wealth effects and the consumption risk of productivity shocks? It may well be possible that volatility in the data is not high enough to generate the mechanism illustrated by Section 3.3. Also, with non-tradables and distribution services, the volatility of the RER may be driven by different channels than stressed in Section 3. Second, our analysis shows that there are two transmission mechanisms through which price volatility can generate low risk sharing: the third moment we include helps discriminating between the two. Note, however, that a negative correlation between the RER and relative output could arise because of strong Balassa–Samuelson effects. Last, we include the fourth moment to mimic the comovements between relative prices and intertemporal trade in the data. This procedure yields an estimate of $\omega$ equal to 0.85. Given the calibrated value of $\mu$, the implied trade price elasticity is below $1/2$, well within the range of available macro estimates discussed above, but at odds with some micro-evidence.

Alternatively, drawing on our theoretical analysis in Section 3.4, we set the trade elasticity equal to 4 based on the estimates in the trade literature, namely, by Bernard et al. (2003). Given the size of the distribution sector, this means that we set $\omega = 8$. Under this calibration, the baseline shock process (reported in Table 2) is not persistent enough to generate strong wealth effects. Hence, following the literature on incomplete markets (e.g. Baxter, 1995), we carry out two sets of experiments. First, we raise the persistence of the process driving tradables productivity shocks to 0.99, while shutting down technological spillovers to keep the process stationary. Second, as a further check to isolate the effects of shock persistence with high elasticity, we use the same aggregate shock process as in Baxter (1995), thus assuming that productivity shocks hit symmetrically both sectors of our economy. The autoregressive coefficient of this aggregate productivity shock is 0.994, implying an annualized value of 0.98, while the variance and covariance matrix is the same as in Backus et al. (1995).

6. WEALTH EFFECTS, RER, AND THE INTERNATIONAL TRANSMISSION OF PRODUCTIVITY SHOCKS

In this section, we analyse the performance of our model and carry out sensitivity analysis. Results for the different calibrations of our economy are shown in Table 3: Table 3 part (a) reports statistics on exchange rates and prices, Table 3 part (b) reports business cycle statistics. In each of these tables, results for our experiments in which $\omega$ is estimated via a method of moments procedure are shown in columns 2 through 5; results for our economy with a high trade elasticity and high shock persistence are shown in columns 6 through 8.

The empirical evidence on the unconditional correlation between international prices and quantities, as well as the their relative volatilities, is summarized by the statistics in the first column of Table 3. The statistics for the data—all filtered using the Hodrick and Prescott...
### TABLE 3
**Method of moments estimation of \( \omega \)**

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Data</th>
<th>Baseline economy</th>
<th>Baseline with taste shocks</th>
<th>Baseline with high elasticity</th>
<th>Persistent tradable shocks</th>
<th>Persistent aggregate shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bond economy ( \omega = 0.85 )</td>
<td>Arrow–Debreu ( \omega = 0.85 )</td>
<td>Bond economy ( \omega = 0.82 )</td>
<td>Arrow–Debreu ( \omega = 0.85 )</td>
<td>Bond economy ( \omega = 8 )</td>
<td>Bond economy ( \omega = 8 )</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>3.90</td>
<td>2.99</td>
<td>0.73</td>
<td>2.94</td>
<td>0.99</td>
<td>1.17</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>1.68</td>
<td>2.42</td>
<td>0.83</td>
<td>2.45</td>
<td>1.07</td>
<td>0.48</td>
</tr>
<tr>
<td>Relative price of non-tradables</td>
<td>0.86</td>
<td>0.77</td>
<td>0.51</td>
<td>0.76</td>
<td>0.48</td>
<td>0.55</td>
</tr>
<tr>
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<tr>
<td>Between real exchange rate and Relative GDPs</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Real exchange rate</td>
<td>-0.33</td>
<td>-0.55</td>
<td>0.87</td>
<td>-0.56</td>
<td>0.11</td>
<td>0.45</td>
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<tr>
<td>Relative GDPs</td>
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<tr>
<td>Relative consumptions</td>
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<td></td>
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<tr>
<td>Real net exports</td>
<td>0.67</td>
<td>0.97</td>
<td>0.63</td>
<td>0.97</td>
<td>0.82</td>
<td>0.99</td>
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<td>Terms of trade</td>
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<td>0.16</td>
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<tr>
<td><strong>(b) Business cycle statistics in the theoretical economies</strong></td>
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<tr>
<td><strong>S.D. relative to GDP</strong></td>
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<tr>
<td>Consumption</td>
<td>0.94</td>
<td>0.48</td>
<td>0.48</td>
<td>0.53</td>
<td>0.67</td>
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<td>Investment</td>
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<td>0.55</td>
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<td><strong>Absolute (in per cent)</strong></td>
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<tr>
<td>Import ratio</td>
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<td>1.62</td>
<td>0.55</td>
<td>1.63</td>
<td>0.81</td>
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<tr>
<td>Real net exports over GDP</td>
<td>0.64</td>
<td>0.17</td>
<td>0.03</td>
<td>0.18</td>
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<td>0.20</td>
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<tr>
<td><strong>Cross-correlations</strong></td>
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</tr>
<tr>
<td>Between foreign and domestic GDP</td>
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</tr>
<tr>
<td>Consumption</td>
<td>0.68</td>
<td>0.38</td>
<td>0.39</td>
<td>0.38</td>
<td>0.33</td>
<td>0.31</td>
</tr>
<tr>
<td>Investment</td>
<td>0.60</td>
<td>0.30</td>
<td>0.37</td>
<td>0.16</td>
<td>-0.01</td>
<td>0.60</td>
</tr>
<tr>
<td>Employment</td>
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<td>0.45</td>
<td>0.45</td>
<td>0.44</td>
<td>0.35</td>
</tr>
<tr>
<td>Between real net exports and GDP</td>
<td>-0.48</td>
<td>-0.38</td>
<td>0.21</td>
<td>-0.39</td>
<td>-0.28</td>
<td>0.22</td>
</tr>
</tbody>
</table>

† The data reported under the heading “Data” are those of the U.S. vis-à-vis the rest of the OECD countries.
filter—are computed with the United States as the Home country and a trade-weighted aggregate of the OECD comprising the European Union, Japan, and Canada as the Foreign country, for the period 1970–2001 (see the appendix for details on the data).

Throughout our numerical exercises, we take a first-order Taylor series expansion around the deterministic steady state and simulate our model economy using King and Watson (1998)’s algorithm. We compute the model’s statistics by logging and filtering the model’s artificial time series using the Hodrick and Prescott filter and averaging moments across 100 simulations. Consistently with the data, in our simulations changes in all real aggregate variables (e.g. GDP) are computed using constant prices (precisely, we use relative steady-state prices).

6.1. Results for the economy with a low trade elasticity

The properties of our economy with the low trade elasticity are described in the second column of Table 3, under the heading “Baseline—Bond economy”. For comparison, the column next to it reports results for the Arrow–Debreu economy, assuming the same elasticity.

A consequence of our calibration strategy is that, with incomplete markets, the RER and the terms of trade are very volatile, the latter even more than in the data. As shown in the second column of Table 3 part (a), the model generates significant volatility in the RER—roughly 75% of that in the data. The volatility of the terms of trade relative to output is 2.42, compared to 1.68 in the data. These results suggest that high volatility of the international prices should not be necessarily taken as a measure of their “disconnect” from fundamentals (see also Chari et al., 2002; and Corsetti, Dedola and Leduc, 2005).

6.1.1. The Backus–Smith correlation and wealth effects. The most novel and important result of our model is that the correlation between relative consumption and the RER is negative, as is in the data. As shown in Table 3 part (a), the correlation generated by the model is −0.24 against our empirical estimate of −0.71. A similar pattern emerges for the terms of trade: its correlation with relative consumption is −0.27, against an empirical estimate of −0.74.

Since under our estimated value of ω the RER and the terms of trade are fairly volatile, our results suggest that a price elasticity that is consistent with the volatility in international prices found in the data also implies a realistically low degree of risk-sharing. What generates a negative Backus–Smith correlation is the mechanism linking exchange-rate volatility to risk-sharing discussed in Section 3. To shed light on the dynamics of this mechanism in a bond economy, we write the Euler equation for international bonds, equating the expected rate of real depreciation to the expected growth rate in marginal utilities of consumption:

$$E_t(\hat{RER}_{t+1} - \hat{RER}_t) \approx E_t[(\hat{U}_{c,t+1}^* - \hat{U}_{c,t}^*) - (\hat{U}_{c,t+1} - \hat{U}_{c,t})] \quad (21)$$

whereas for simplicity we abstract from (negligible movements in) the endogenous discount factor. By trading bonds, agents ensure that, in expectations, real depreciation is associated with higher consumption growth in the domestic economy relative to consumption growth abroad (precisely, lower relative growth in the marginal utility of consumption). Now, to the extent that the tight link between consumption growth rates is inherited by consumption levels, the

20. In addition, we have solved the model using a second-order Taylor expansion of the model’s equilibrium solution based on the algorithm Dynare, developed by Michel Juillard. Results from this exercise are very similar to the ones reported in the text. To check the numerical accuracy of our results, we carried out the test developed by Den Haan and Marcet (1994), finding that the model’s solution passes their accuracy test.

21. We still find that the RER is more volatile than the terms of trade even when we set ω just to match the volatility of the terms of trade or drop the volatility of the RER from the moments in the estimation. Moreover, the model can still generate the negative transmission mechanism and the lack of international consumption risk sharing.
above expression suggests that international borrowing and lending tends to make the correlation between relative consumption and the RER positive. But in a stochastic environment, the international bond is traded only after the resolution of uncertainty and does not provide households with ex-ante insurance against country-specific income shocks—it only makes it possible to re-allocate wealth and smooth consumption over time. If unexpected shocks have large wealth effects, relative consumption and the RER can move in the opposite direction on impact. Indeed, under our calibration, conditional on a productivity shock to tradables, the RER and relative consumption comove negatively on impact, but positively in the aftermath of the shock, when their joint dynamics is dictated by the equation above. For this reason, the (unconditional) Backus–Smith correlation in a bond economy is less negative than under financial autarky. It will also become higher and closer to that implied by complete markets, the weaker the impact response (in absolute value) of the RER—that is, the less volatile the RER and the terms of trade on impact.

As emphasized by the above analysis as well as by our results in Section 3, standard international business cycle models with incomplete markets can account for a negative Backus–Smith correlation, provided the equilibrium dynamic response to productivity shocks generate strong endogenous wealth effects. Now, using our model, we can actually quantify the wealth effects associated with a productivity increase in the Home tradables on Home and Foreign consumptions, as well as hours worked, for different asset market structures. For ease of exposition, we abstract from spillover effects.

In carrying out this exercise, we find that, if markets are complete, a positive productivity shock in Home tradables brings about positive wealth effects on Home and Foreign consumptions. The wealth effect on Foreign consumption is positive, since the increase in Home output is shared with households abroad. The wealth effect on Home and Foreign hours is negative: other things equal, richer households would prefer to supply less hours to the market. Assuming incomplete markets, however, does not automatically change the above picture. Indeed, if we set the trade elasticity close to 1 (as is the case in our model when there are no distribution services, $\eta = 0$, and $\omega = 0.85$), wealth effects in the bond economy and the complete-market economy are very similar to each other. This result is in line with previous findings in the literature and our analysis in Section 3—especially in light of Cole and Obstfeld (1991).

Wealth effects however diverge substantially when the trade elasticity is relatively low. Reflecting the improvement in the terms of trade following an increase in the productivity of Home tradables, the wealth effects on Home consumption and hours worked roughly double relative to the Arrow–Debreu economy. Notably, the wealth effects on Foreign consumption and hours have the opposite sign.

The transmission mechanism in our economy has a number of relevant implications for the quantitative properties of our model. As is well known, most open-economy models—including those allowing for nominal rigidities and monetary shocks—predict a strong and positive link between relative output and RER. As Stockman (1998) points out, this prediction is at variance with the evidence: the empirical correlation shown in Table 3 part (a) is $−0.19$. A similar shortcoming concerns the correlation between relative output and the terms of trade, which is negative in the data (and equal to $−0.33$), while it tends to be positive in quantitative models. In contrast,
in our baseline economy with incomplete markets the correlation between relative output and the RER (the terms of trade) is negative—equal to $-0.54$. This is because, as emphasized in Section 3, positive productivity shocks in the tradable sector appreciate the terms of trade and the RER.\textsuperscript{25} For this very reason, the model can also match the observed positive correlation between international relative prices and real net exports, shown at the bottom of Table 3 part (a).\textsuperscript{26}

### 6.1.2. The RER and the terms of trade.

A remarkable result in Table 3 part (a) is that our model is consistent with the empirical ranking of variability in international prices: the U.S. RER is more volatile than the U.S. terms of trade. To appreciate this result fully, consider the following expression for the RER in our model, written in log-linear form:

$$\text{RER}_t = (1 - \mu)(2a_H - 1)\tilde{r}_t + \mu(\hat{P}_{N,t}^* - \hat{P}_{N,t}) + \Omega(\hat{q}_t^* - \hat{q}_t),$$

where $0 < \Omega < 1$ and $\hat{q}_t^*$ represents the relative price of non-tradables.\textsuperscript{27} The expression above decomposes the movements of RER into movements in the terms of trade and in the relative price of non-traded goods. This equation makes it clear that, if $\mu = 0$ and $\Omega = 0$—in other words, absent deviations from the law of one price due to distribution services (which drive a wedge between the terms of trade and relative prices at the consumer level) and absent movements in the price of non-tradables across countries—our model would not replicate the empirical ranking of volatility discussed above. This points to a clear shortcoming of models that assume that all goods are perfectly tradable and that disregard deviations from the law of one price at the consumer level (see Heathcote and Perri, 2002). With $\mu \neq 0$ and $\Omega \neq 0$, the real exchange rate also responds to fluctuations in $\hat{P}_{N,t}^* - \hat{P}_{N,t}$ and $\hat{q}_t^* - \hat{q}_t$, whose quantitative importance depends, among other factors, on the extent to which shocks are correlated across countries and sectors.\textsuperscript{28}

In our model, the real exchange rate and the terms of trade are tightly related. Their correlation is positive, though higher than in the data (equal to 0.99 against 0.52). A positive sign for this correlation is an important result relative to alternative models that—like ours—allow for deviations from the law of one price, but do so by assuming sticky prices in the importer’s currency. As argued by Obstfeld and Rogoff (2000), these models can generate high exchange rate volatility as well, but at the cost of inducing a counterfactual negative correlation between the RER and the terms of trade.

Also broadly consistent with the empirical evidence are the behaviours of the relative price of non-tradables and the channels through which the RER fluctuates in our model. First, as shown in Table 3 part (a), the volatility of the relative price of non-tradables (relative to output) predicted by our model is 0.77. The corresponding empirical estimate is 0.86. Second, the ratio between the S.D. of the relative price of non-tradables across countries and the S.D. of the RER is 45%.

\textsuperscript{25} The impulse responses to a technology shock to tradables are analysed in details in Corsetti et al. (2004).

\textsuperscript{26} In the tables, net exports (in deviations from steady state) are measured in real terms using steady-state prices as $(\text{net export/y})_{\text{real}} = \frac{\bar{P}_t C_{H,y} - \bar{P}_t C_{F,y}}{y}$.

\textsuperscript{27} Namely, $\Omega = \alpha N \bar{q}^{\phi/(\phi - 1)} / (\delta_T + \alpha N \bar{q}^{\phi/(\phi - 1)}) > 0$, where $\bar{q}$ denotes a steady-state value, and $\frac{1}{1 - \phi}$ is the elasticity of substitution between tradables and non-tradables.

\textsuperscript{28} Our measured TFP shocks have a relatively high sectoral correlation, equal to 0.47 within a country. This value is virtually identical to the one estimated in Stockman and Tesar (1995) using a slightly different methodology and a different time span. While imperfect correlation across sectoral TFP shocks is a key ingredient for our results, an important role is also played by the international transmission. Specifically, the ranking of price volatilities changes across trade elasticities and asset markets structures, as can be seen by comparing results in our baseline bond economy and the Arrow–Debreu economy in Table 3.
Using a weighted average of U.S. bilateral RER, Betts and Kehoe (2001) estimates that this ratio is between 35% and 44%.29

6.1.3. International business cycles. We conclude the analysis of our model by discussing a set of findings concerning business cycle statistics, shown in Table 3 part (b). In the baseline bond economy (second column in the table), the cross-country correlations of GDP, consumption, investment, and hours are all positive, consistent with the evidence. Positive comovements in production mainly arise because of the positive correlation of shock innovations across countries—namely, they are also predicted by our Arrow–Debreu economy. However, in the bond economy, these comovements are strengthened by the negative transmission mechanism—whereby the appreciation of the Home terms of trade in response to positive domestic productivity shocks transpires into large negative wealth effects abroad, raising labour efforts and thus employment and production in the Foreign economy. The contribution of the negative transmission is less apparent at the aggregate level: the cross-correlation of GDP is essentially identical in the complete and the incomplete market economy. It is instead more appreciable at sectoral level: the cross-correlation of traded (non-traded) output rises from 0.43 (0.33) in the Arrow–Debreu economy to 0.51 (0.36) under incomplete markets.

In spite of the negative transmission mechanism, the cross-border correlation of consumption is positive. The source of this result is to be found in the estimated productivity spillovers, which reduce the negative effects on Foreign wealth from Home terms of trade movements. Removing spillovers implies that the cross-border correlation of consumption becomes negative—with limited or no impact on the cross-country correlation of production, employment, and investment.

Consumption is less cross-correlated than output across countries, matching the ranking in the data. As is well known, international business cycle models typically yield the opposite prediction, independently of the cross-border correlation of productivity shocks—Backus et al. (1995) dub this empirical incongruity the “quantity anomaly”. Observe that our model does not suffer from this anomaly, regardless of whether financial markets are complete (the ranking of cross-correlations is correct also in the Arrow–Debreu economy). Distribution services appear to play an important role for this result since a correlated increase in the consumption of tradables across borders triggers an increase in the production of non-tradables (required to provide distribution services) in both countries. Indeed, absent a distribution sector, the model predicts that the cross-border correlation of consumption is higher than output (see the sixth column, labelled “no distribution”, in the table).

Our economy displays some discrepancies with the empirical evidence which are however common to standard international business cycle models (both one-sector models such as Backus et al. (1995) and two-sector models such as Stockman and Tesar (1995)). Namely, consumption, investment, and employment are slightly less volatile—relative to output—than in the data; net exports are also substantially less volatile than in the data. Nonetheless, as can be seen from comparing our baseline with the Arrow–Debreu economy, significant wealth effects in incomplete markets turn out to be somewhat helpful in improving the predicted volatility of trade quantities, like the import ratio.

6.1.4. Further experiments. We now assess the sensitivity of our results to (a) removing the distribution sector; (b) using different specifications of investment; (c) introducing taste

29. Following a different procedure, Engel (1999) finds that deviations from the law of one price in traded goods virtually account for all the volatility of the U.S. RER. However, using a methodology similar to Engel’s and Burstein, Eichenbaum and Rebelo (2005) find that this proportion falls below 50% when traded goods prices are proxied with import and export prices.

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shocks; and (d) removing cross-country spillovers from the process driving productivity shocks. In each of these cases, we re-calibrate \( \omega \) to match the four moments discussed in the calibration section above. The complete set of results is reported in the web appendix.

Changing the distribution margin and the elasticity of substitution. When we abstract from distributive trade, and set \( \eta = 0 \), the value of \( \omega \) for which the model matches the chosen moments is 0.16, a good deal lower than in our baseline economy. With a lower elasticity of substitution, but no distribution services, the model succeeds in lowering the Backus–Smith correlation: the RER and relative consumption are barely correlated in this case. The underlying mechanism has already been thoroughly analysed in Sections 3.

However, as discussed in Section 5, the need to combine tradables with retailing makes the price elasticity of imports lower than the value implied by the preference parameter \( \omega \). Excluding distribution, fitting the volatility of international prices requires the Home and Foreign goods to be poor substitutes in agents’ preferences.

Moreover, with \( \eta = 0 \), there are no deviations from the law of one price, so that movements in the RER are not consistent with an important stylized fact of the international economy. Specifically, movements in the relative price of non-tradables across countries contribute to RER fluctuations much more than in the economy with a retail sector. The S.D. of the relative price of non-tradables across countries is now 88% of that of the RER, a significantly higher fraction than in the data. By the same token, the relative price of non-tradables is about 80% more volatile than in the data (1.53 against 0.86).

Although these results show that distribution services are not necessary to account for the Backus–Smith puzzle, they also suggest important advantages of specifying them in our quantitative model. First, the low trade elasticities typically estimated by the macro literature need not imply too strong restrictions on agents’ preferences. Second, because of distribution, our model generates low international risk sharing while being in line with important empirical facts underlying the movements in RER. Since the RER is a crucial factor in the Backus–Smith puzzle, we view this advantage as a significant one.

Changing the investment specification. In our baseline setting investment is carried out solely in domestically produced tradable goods. Here, we allow for a more general specification in which investment is a composite good comprising both Home and Foreign tradables. We assume that investment goods are given by the following CES aggregator

\[
I_{T,t}(j) \equiv \left[ a_{H}^{1-\rho} I_{H,t}(j)^{\rho} + a_{F}^{1-\rho} I_{F,t}(j)^{\rho} \right]^{1/\rho},
\]

where \( I_{H,t} \) (\( I_{F,t} \)) is the level of investment in terms of the domestic (imported)-traded good. As in our baseline calibration, we set \( a_{H} \) and \( a_{F} \) such that imports (which now also include investment) are 5% of aggregate output in steady state. This exercise introduces imports of investment goods—which can be viewed as intermediate goods. In line with the evidence, we then model distribution services in the investment sector setting the share of distribution services in the price of investment to be 16-7%, a figure estimated by Burstein, Neves and Rebelo (2004).

With the more general CES specification for investment, the values of \( \omega \) needed to reproduce the volatility of the RER relative to that of output are smaller than under our baseline specification for investment. This is essentially for the same reason discussed in our first sensitivity exercise above: investment goods can now be imported from abroad, and investment does not require as much distribution services as consumer goods do. Thus, any given price elasticity of imports corresponds to a lower elasticity of substitution relative to our baseline specification (see also Section 5).
Nonetheless, the model still succeeds in generating a significant departure from the complete markets outcome. Although the RER and relative consumption are not as negatively correlated as in our previous experiments, their correlation remains well below 0. When $\omega = 0.5$, the model still predicts a negative Backus–Smith correlation of $-0.13$.

**Taste shocks.** The third and fourth columns of Table 3 report results from introducing taste shocks both in the bond economy, for which we recalibrate $\omega$ according to our method of moments, and in an economy whereas we allow trade in a complete set of Arrow–Debreu securities (in this case, we kept the value of $\omega$ equal to 0.85). In these experiments, we modify (17) as follows:

$$U[C_t, \ell_t] = \frac{\kappa_t C_t^\alpha (1 - \ell_t)^{1-\alpha} [1 - \sigma]}{1 - \sigma} - 1,$$

where $\kappa_t$ is the taste shock. Following Stockman and Tesar (1995) we calibrate $\kappa_t$ assuming that taste shocks are uncorrelated across countries and have a S.D. and serial correlation equal to the largest between the two productivity shocks, 0.0089 and 0.961, respectively (see Table 2). We stress two main results.

On the one hand, our findings appear broadly unchanged after the introduction of tastes shocks (see the fourth column of Table 3). In the bond economy, wealth effects driven by productivity shocks keep playing a crucial role in explaining lack of risk sharing—a result which is largely independent of taste shocks.

On the other hand, the “fit” of the complete-market version of our model substantially improves following the inclusion of preference shocks that are as volatile and persistent as productivity shocks (see the fifth column of Table 3). The introduction of taste shocks in international business cycle models with complete markets weakens the links between relative consumption and relative marginal utility, thus being functional to generating a low or negative correlation between RER and relative consumption. Indeed, the correlation between the RER and relative consumption is equal to $-0.29$; that between the terms of trade and relative consumption is negative as well, equal to $-0.54$. Moreover, the terms of trade and the RER are now positively correlated (0.59), slightly more than in the data; both comove positively with net exports. In the Arrow–Debreu economy, taste shocks also raise the volatilities of the RER, the terms of trade, the import ratio and net exports, which however remain much lower than in the data. The cross-correlation of consumption also becomes counterfactually negative.30

Observe that the improvement in the performance of the complete-market version of our model following the introduction of taste shocks lends strong support to one of the main tenets of this paper. Namely, the basic mechanism underlying the Backus–Smith puzzle requires strong demand effects arising from shocks to fundamentals. The results shown in the fifth column of Table 3 point to the logical possibility that the received interpretation of the evidence in support of a low degree of international risk sharing—including the Backus–Smith puzzle discussed in Section 2 but also the evidence on limited international portfolio diversification—is mistaken. That is, they suggest that the Backus–Smith correlation could be negative in the data in spite of a very high degree of cross-country consumption insurance, due to exogenous shocks to preferences—that is, demand—which modify the sign of the correlation between marginal utility of consumption and consumption itself. The contribution of our paper, instead, is to show that there is a viable alternative to assuming exogenous demand shocks, more in line with the widespread interpretation of the evidence on lack of risk sharing. In our economy with incomplete asset markets,
strong demand and wealth effects follow in equilibrium from supply disturbances only. Adding
taste shocks to our economy, as discussed above, is utterly inconsequential.31

6.2. Results for the economy with high trade elasticity and shock persistence

With high trade elasticities, our analytical results in Section 3 suggest that the Backus–Smith
puzzle can be matched if shocks are very persistent. How persistent should productivity shocks
be? We address this issue by carrying out a quantitative experiment where we set the trade elastic-
ity equal to 4 (ω = 8), in line with micro-estimates, and let the autoregressive coefficient for the
process driving tradables productivity shocks approach a unit root. Results from this experiment
are shown in the last three columns of Table 3.

6.2.1. The Backus–Smith correlation and wealth effects. The sixth column of Table 3,
under the heading “Baseline With High Trade Elasticity”, assumes that sectoral productivity
shocks follow the estimated process in Table 2. Clearly, this process is not persistent enough:
the correlation between the RER and relative consumption in the bond economy is 0.73. This
is somewhat lower than in the Arrow–Debreu economy, but still positive and quite high. The
results of increasing the persistence of shocks is shown in the table under the heading “Persistent
shocks”—whereby the autocorrelation coefficient of shocks to tradables is raised from 0.82 up
to 0.99.

With highly persistent shocks, the model generates a Backus–Smith correlation equal to
−0.12.32 The insights from the analysis of the simple endowment economy in Section 3.4 shed
light on the reasons for this result. In Section 3.4, we posited that the domestic economy experi-
ences a gradual increase in (relative) tradable output, driven by an exogenously specified supply
process with permanent effects. Anticipating such dynamics, agents smooth consumption by rais-
ing demand above output in the short run, causing a temporary appreciation of the terms of trade
and the RER. The same gradual increase in relative tradable output (and the associated strong
response of domestic demand on impact) also occurs in our quantitative analysis in this section.
However, what now drives the hump-shaped response of output to productivity is the endoge-
nous dynamics of capital accumulation and labour supply brought about by the autoregressive
but highly persistent productivity process. Output grows over time and peaks a few periods af-
after the shock. While such a peak is temporary, a high degree of shock persistence can generate
wealth effects that are large enough to cause the terms of trade to appreciate on impact. For
these reasons, and similarly to the baseline bond economy in the previous subsection, the terms
of trade are negatively correlated with relative output and relative consumption, in line with
the data. Precisely, the increase in relative wealth—unanticipated before the shock hits—breaks
again the tight expected link between consumption and RER dynamics implied by equation
(21) above.

With high shock persistence, a relatively high trade elasticity is a crucial condition to obtain
the dynamic effects of productivity gains described above. Otherwise, the increase in the world
supply of Home tradables over time would generate a substantial drop in their prices—which
would in turn reduce the international value of Home output and hence Home wealth. In the
endowment economy of Section 3.4, we show that a necessary condition for a negative Backus–
Smith correlation is a value of the trade elasticity larger than one—the required value depending
on the relative strength of the short-run and long-run effects of the shock. The same is true

31. In the working paper version (Corsetti et al., 2004) we also analyse whether the Backus–Smith anomaly could
be accounted for by Balassa–Samuelson effects, linking exchange-rate fluctuations to movements in the relative price
of non-tradables, with negative results.

32. This correlation is still as high as 0.6 when the autoregressive coefficient is set equal to 0.90, and falls to 0.04
when this coefficient is set to 0.98.
FIGURE 2
Trade elasticity and Backus–Smith puzzle with highly persistent shocks

in our quantitative model, as shown in Figure 2a, which plots the correlation between relative consumption and the RER, together with the correlation between relative aggregate output and the terms of trade, all HP-filtered, for values of spanning trade elasticities between 1 and 5. A negative Backus–Smith correlation emerges for a trade elasticity above 2, that is, above the values usually adopted by the macro quantitative literature. The correlation between terms of trade and relative aggregate output is instead negative already for a trade elasticity above 1.5.

The role of trade elasticities can be further illustrated by computing wealth effects in response to a productivity shock to Home tradables with an autoregressive coefficient equal to 0.99, in economies with different asset market structures and trade elasticities, following the same methodology as in Section 6.1. With a trade elasticity equal to 4 (corresponding to $\omega = 8$ and $\eta = 0.5$), we find apparent and sizeable differences between the complete and incomplete market economies. Relative to the case of complete markets, with imperfect risk sharing Foreign agents benefit much less in terms of consumption, as well as in terms of hours worked. Nonetheless, the sign of the consumption and leisure spillover remains positive overall, since the impact

33. For the reasons thoroughly discussed in the rest of the paper (see Sections 3.1, 3.2, and 6.1), the Backus–Smith correlation also turns negative for low values of the trade elasticity—around 0.5.
appreciation of the terms of trade is not very large. In contrast, differences in wealth effects become much smaller with a trade elasticity equal to 1.5.

6.2.2. RER and international business cycles. The business cycle properties of the model under high elasticity and high shock persistence are largely comparable to the ones of the baseline bond economy with a low elasticity (compare the second and the seventh columns of Table 3), once one takes into account the absence of spillovers. Because of the high trade elasticity, however, the model performs less satisfactorily in generating volatile international prices. Under this calibration of ω with persistent shocks, the bond economy yields terms of trade and an RER that are roughly 40% as volatile as their empirical counterparts. Observe that this is the only large difference between the two economies.34

6.2.3. Further experiments. As a further check, we carry out a quantitative experiment using the same aggregate productivity shock process as in Baxter (1995), which features an annualized autoregressive coefficient equal to 0.98. For this process, Figure 2b shows that the Backus–Smith correlation changes sign for values of the trade elasticity slightly lower than in the case of persistent shocks to tradables—the countervailing Balassa–Samuelson effect of shocks to non-tradables is now absent. Conversely, the correlation between the terms of trade and relative output turns negative for higher values of the elasticity than in Figure 2(b), because aggregate shocks are slightly less persistent than tradable shocks (0.98 against 0.99).

Second moments with the elasticity equal to 4 are shown in the last column of Table 3, under the heading “Persistent Aggregate Shocks”. As in our previous experiments, the model generates a negative Backus–Smith correlation, equal to −0.67—quite close to that in the data. Remarkably, all statistics are broadly in line with our baseline economy of Section 6.1, with some important deviations. Abstracting from differences between tradables and non-tradables productivity dynamics causes the volatility of international prices and non-traded goods prices to fall counterfactually, and considerably, as well as the RER and the terms of trade to have roughly the same volatility.

These results suggest that, for values of the trade elasticity close to those found in micro-studies, the Backus–Smith correlation can be matched even in one-sector models without non-tradables (e.g. as Backus et al., 1995), although the predicted behaviour of RER and terms of trade is less satisfactory than in two-sector models with non-tradables (e.g. as Stockman and Tesar, 1995).

The main message of the analysis in this section is broadly in line with the findings of Baxter and Crucini (1995) in the context of a one-good model, namely, that the scope for self-insurance via intertemporal trade is limited when shocks are close to have a unit root. In this respect, our exercise can be read as a generalization of these authors’ result to an environment in which the trade elasticity is high but finite (goods are not homogenous). However, our analysis also yields an important novel result: with highly persistent shocks, a sufficiently high trade elasticity is key for the model to account for large wealth effects and relative price changes consistent with the Backus–Smith evidence.

7. CONCLUDING REMARKS

A large body of empirical literature provides evidence suggesting that the degree of international consumption risk sharing is quite limited, and cross-border financial markets are not

34. The fact that the cross-correlation of consumption is close to 0, reflects the negative transmission mechanism in the absence of spillovers (see our discussion above). Observe that in the Arrow–Debreu economy, this cross-correlation is positive because of full risk sharing.
developed enough to replicate the paradigm of complete markets. This literature raises important conceptual and theoretical issues in modelling the international transmission of real and nominal shocks. Specifically, it emphasizes the need for international business cycle models to account for possible large, uninsurable wedges between domestic and foreign wealth following shocks to fundamentals—driving the equilibrium allocation significantly away from the complete-market benchmark.

In this paper, we focus on the Backus–Smith puzzle as a crucial dimension to assess the performance of business cycle models. Reconsidering the core transmission mechanism in standard international business cycle models with incomplete markets, we show, analytically and quantitatively, that productivity shocks can result into large wealth, and hence demand, effects.

We identify two sets of conditions for this result. The first emphasizes a low trade elasticity, associated with high volatility of international prices and thus strong wealth effects from price changes. The second points to high shock persistence, consistent with the literature on incomplete markets. As is well known, trade elasticities are the subject of an ongoing debate at both empirical and quantitative levels: the uncertainty about these values in the literature is quite broad, with aggregate and short-run estimates being substantially lower than disaggregated and long-run estimates. When we estimate this parameter in our model using a method of moments procedure, we find a value in line with the range of estimates from empirical macro studies.

Nonetheless, if one subscribes to the view that trade elasticities are as high as those estimated in the trade literature, our analysis suggests a strategy to make sense of the low degree of risk sharing in the data. This consists of finding endogenous transmission mechanisms which, similar to the case of high shock persistence analysed in our paper, bring about a hump-shaped response of output to technology shocks (thus inducing a positive autocorrelation coefficient in output growth). Intriguingly, Cogley and Nason (1995) have argued that the latter feature is an important stylized fact of the U.S. economy that standard business cycle models have a difficult time accounting for.

We conclude by stressing an important implication of our study for empirical and policy research. A number of contributions to the literature have emphasized terms of trade adjustment in response to country-specific supply shocks, as a potential mechanism providing risk insurance to countries specialized in the production of different types of goods. This paper stresses that the Backus–Smith evidence is at odds with such a positive view of the international transmission mechanism. Specifically, to account for the negative correlation between relative consumption and the RER observed in the data, the terms of trade response to productivity shocks needs to magnify, rather than smooth, the consumption risk implied by fluctuations in output. Hence, positive supply shocks are matched on impact by stronger terms of trade, widening the wedge between domestic and foreign wealth.

Overall, then, our analysis suggests that equilibrium terms-of-trade movements may be much less effective in containing divergences in wealth and consumption in a world where the speed of technical progress is not even across countries. On the contrary, to the extent that financial markets are not well developed, relative price movements can even be counterproductive, in the sense of amplifying cross-country wealth differences. This issue is crucial to our understanding of the international transmission mechanism, and underlies much of the ongoing work on the gains from international policy coordination.

APPENDIX. DATA SOURCES

This appendix describes the annual data used in this paper. Real GDP, the GDP deflator, export and import prices (in local currency units), real private consumption, real private fixed investment, total employment were taken from the OECD’s Outlook database, for the period 1970–2001. Since the OECD gathers data from national statistical agencies, U.S. real...
REFERENCES


